



# Reservoir Computing: concepts and hardware implementation in photonic

Laurent Larger<sup>1</sup>

<sup>1</sup> FEMTO-ST/ Optics, UMR CNRS 6174  
University Bourgogne Franche-Comté

5-8, May 2015 / St-Paul de Vence, France  
Colloque du GDR BioComp





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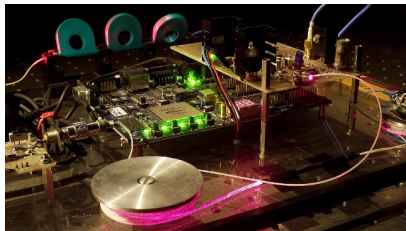


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# Outline

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1. Introduction, background, motivations & basics
2. RC: Where does it come from?
3. Important concepts in RC
4. Delay dynamics: a bit of theory
  - Basics in NL delay dynamics
  - Space-Time analogy, ex. of Chimera states
5. Photonic implementations of RC
  - Photonic delay-based RC for spoken digit recognition
6. Conclusions



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Important concepts in RC

Delay dynamics: a bit of theory

Photonic implementations of RC

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# RC: already a contest winner

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**Successful achievements of computer simulated RC:**



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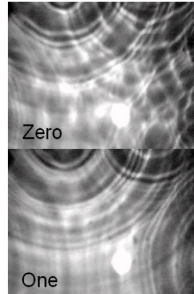
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[Fernando, Sojakka, '03]

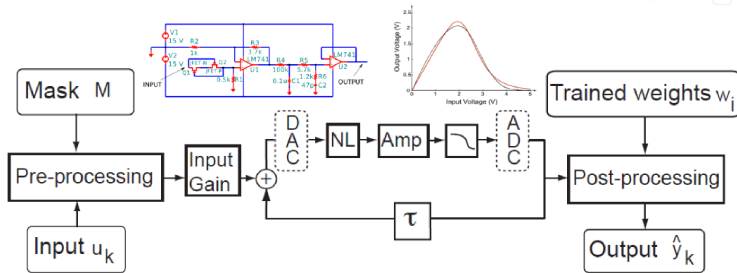


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- Bucket of liquid

Fernando & Sojakka, "Advances in Artificial Life", pp.588-597 (2003, Springer)

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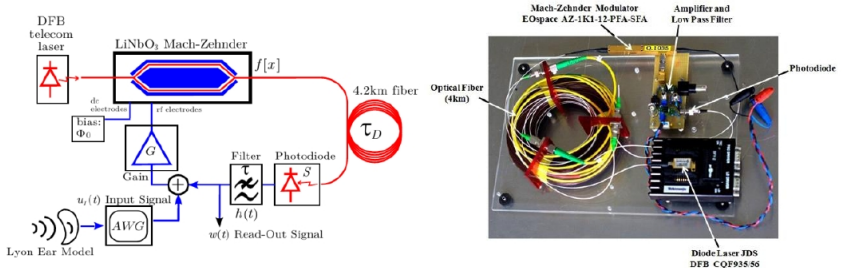


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Appeltant *et al.*, *Nature Commun.* 2:468 (2011)

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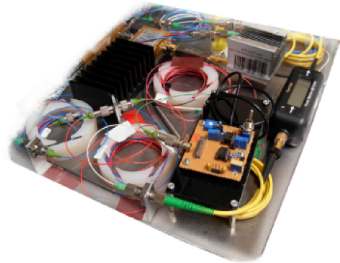
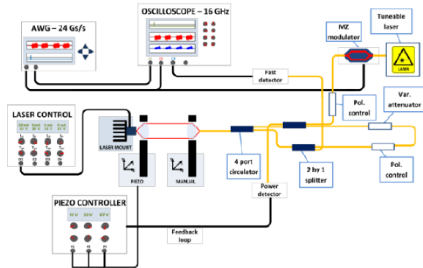


## ...and now even available in hardware

- Bucket of liquid
- Low speed analogue electronic
- Moderate speed optoelectronic

Larger *et al.*, *Opt.Expr.* **20**(3) 3241. Paquot *et al.*, *Sci.Rep.* **2**:287. Martinenghi *et al.*, *Phys.Rev.Lett.* **108** 244101. (2012)

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## ...and now even available in hardware

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- Moderate speed optoelectronic
- High speed all-optical and optoelectronic demo

Brunner *et al.*, *Nature Comm.* 4:1364. Jacquot *et al.*, *CLEO Europe*. (2013)

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RC: Where does it come from?

Important concepts in RC

Delay dynamics: a bit of theory

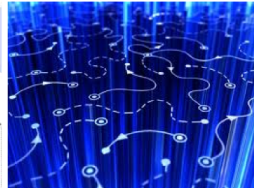
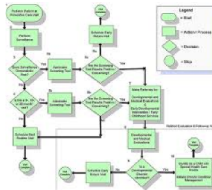
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# RC: Where does it come from?

## Conceptual viewpoint: from rules to controlled freedom

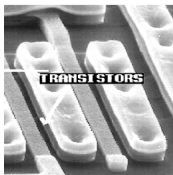
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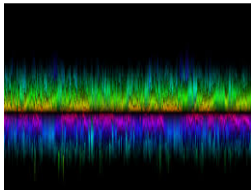
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# RC: Where does it come from?

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- Conventional computing (Binary digIT, logic gates)
- Digital computers & algorithms, more and more complex

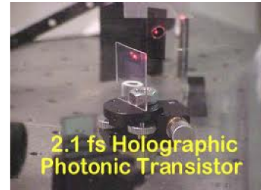
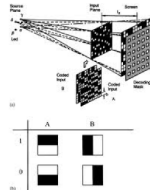
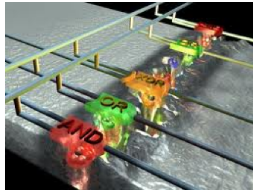




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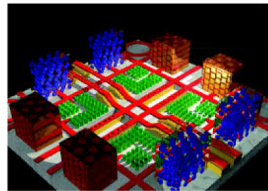
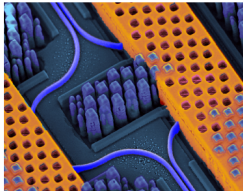
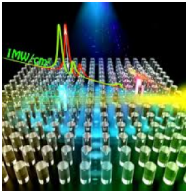
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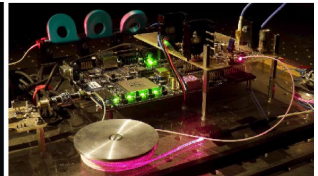
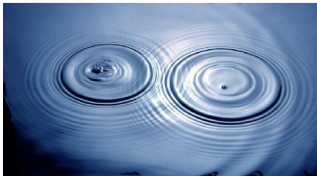
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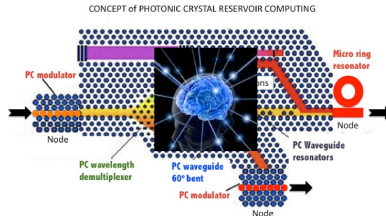
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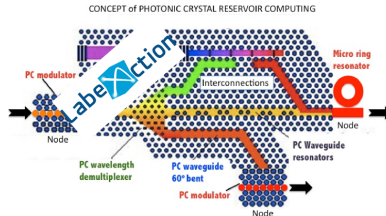
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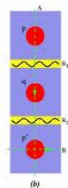
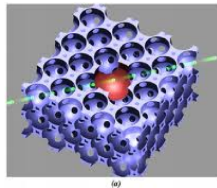
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- Conventional computing (Binary digIT, logic gates)
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- Beyond “Turing-Von Neumann” viewpoint: RC, bio-inspired
- ... and quantum optical computing (*not -yet- connected*)



# RC: Where does it come from?

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## Historical viewpoint, dates

- 1995→ basic RC principles (P.F. Dominey, mammalian brains)
- 2000→ intern. patent applications (Fraunhofer IAIS, granted 2010)
- 2001→ ESNs and LSMs (Trieste; Jaeger & Maass)
- 2004→ RC group at Univ. of Gent (B. Schrauwen)
- 2005→ ESN special session at IJCNN 2005 (J. Principe)
- 2006→ ESN + LSM workshop at NIPS (Maass & Jaeger)
- 2007→ Special RC issue, *Neural Networks* (Jaeger, Maass, Principe)
- 2007→ Special session on RC at ESANN (Schrauwen)
- 2008→ FP7 STREP “Organic”: RC for speech recognition
- 2009→ FP7 STREP “Phocus”: RC for photonic computation  
FP7 IP “Amarsi”: biologically inspired robot motor control
- 2012→ RC workshop at ECCS, Brussels (Massar, Schrauwen, Fischer)
- 2013→ RC workshop, Labex ACTION, DEMO 3, Besançon

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Introduction, background, motivations & basics

RC: Where does it come from?

Important concepts in RC

Delay dynamics: a bit of theory

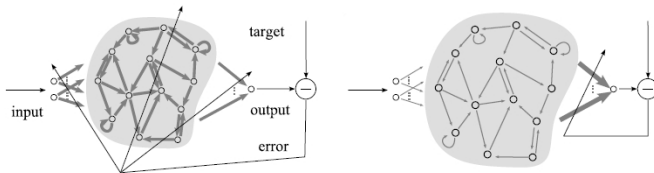
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# Basics in Reservoir Computing

## Foundation of the RC concept: Recurrent Neural Network (RNN, left; right: RC)

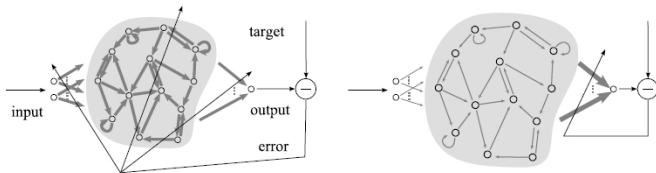


- “Randomly” **fixed** internal network connectivity

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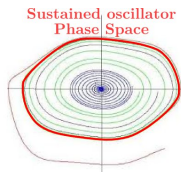


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- Train how to **Read** the Reservoir response (only, bold arrows)

# Basics in Reservoir Computing

## Foundation of the RC concept:

*Asymptotic vs.  
Transient dynamics  
(huge space for transients  
out of the stable solution)*

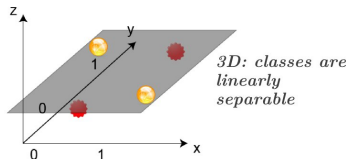
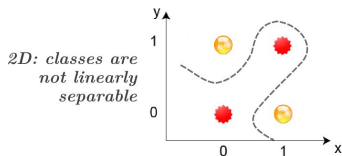


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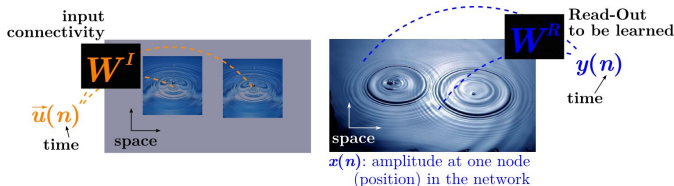


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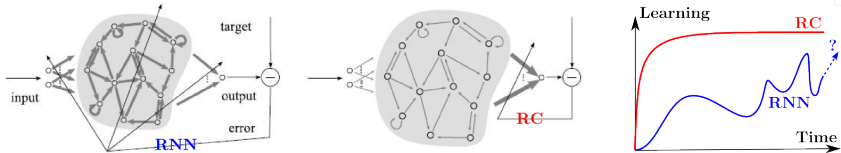
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- Essential feature: **dynamic** (not static). *Nonlinear transient computing*
- **Complexity, dimensionality**
- **Input** triggers a **transient**, which (linear) **Read-Out**  $W^R$  is to be **learned**, via e.g. one simple Matlab code line ( $W_{\text{opt}}^R = Y_{\text{target}} X^T (X X^T - \lambda I)^{-1}$ )

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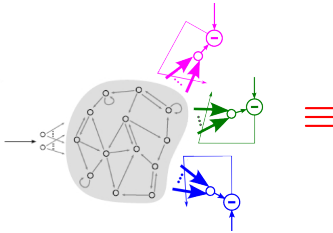
# RC Breakthrough: simple & efficient



## Breakthrough contributions of RC in RNN

- Speed-up & simplify the training, without computational power loss!

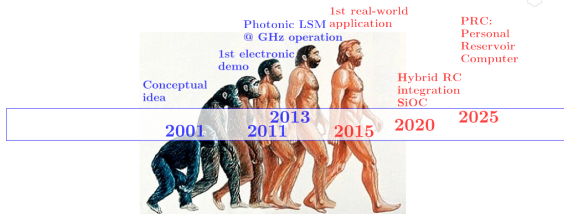
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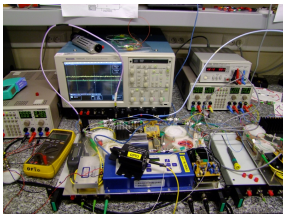
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## Breakthrough contributions of RC in RNN

- Speed-up & simplify the training, without computational power loss!
- Can learn simultaneous multi-tasking (same input & Reservoir)
- Already efficient, and considerable scope for improvement
- Dedicated hardware implementation demonstrated

# Drawback of RC

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***RC: A field “under active construction...”***

- “Black box”, but theoretical description in constant progress

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- An annoying “simpler & better” reputation
- Nonlinear dynamics (one of the main theoretical background of RC) poorly taught, low popularity in engineering education programs



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- An annoying “simpler & better” reputation
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- Difficult to get attention on analogue brain inspired computing concept, in the golden age of digital computers

# Implementation: Which kind of Reservoir?

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*How to design a physical hardware fulfilling the role of a neural network?*

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- Should own appropriate dynamical properties (fading memory, approximation property, separation property)
- Should allow for suitable connectivity within the Reservoir
- ... One possible solution for the Reservoir: **Delay dynamics**

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- Basics in NL delay dynamics

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# How often can we experience NLDDE?

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**Actually every day, everywhere!**

- Living systems (population dynamics, blood cell regulation mechanisms, people reaction after perception and neural system processing, . . . )



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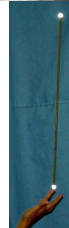
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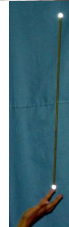




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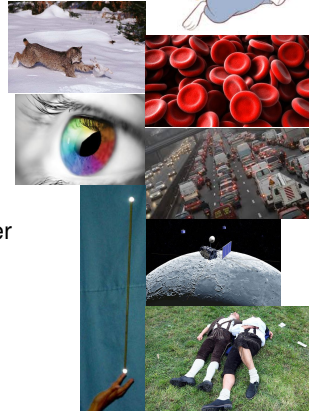
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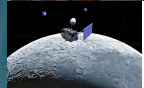
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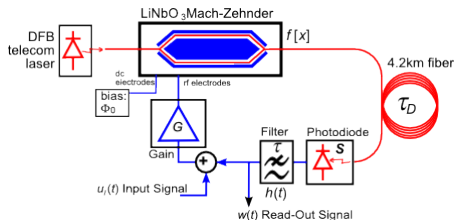
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- Human stand-up position control (and effects of increased perception delay after alchoolic drinks)
- Hot and cold oscillations at shower start



**Any time information transport occurs (at finite speed), resulting in longer propagation time compared to intrinsic dynamical time scales**

# RC with nonlinear delay dynamics

## Paradigmatic Optoelectronic setup

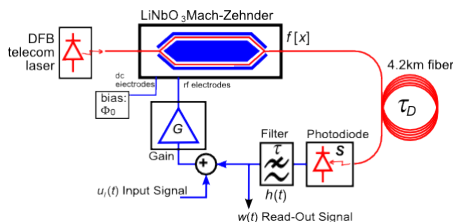


- Already successfully used for optical chaos communications

Argyris *et al.*, *Nature*, **436** 343-346 (2005); Larger and Dudley, "Optoelectronic Chaos", *Nature* **465** 41-42 (2010)

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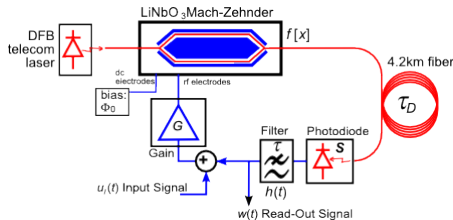


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Yao and Maleki, *Electron..Lett.* **30**:18 1525 (1994)

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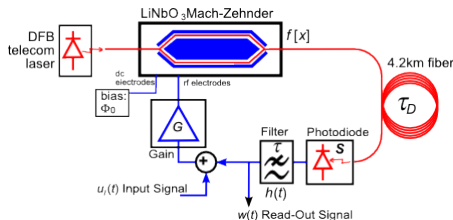


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- More recently, served as the experimental basis for the two first demonstration of photonic RC

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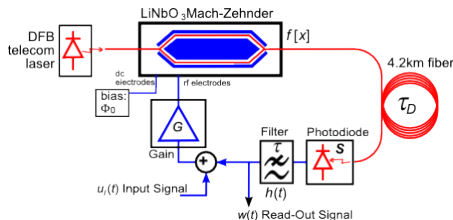


- Already successfully used for optical chaos communications
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- 1<sup>st</sup> electronic demonstrator based on a similar delay dynamics

Appellant *et al.*, *Nature Comm.* 2:468 (2011)

# RC with nonlinear delay dynamics

## Paradigmatic Optoelectronic setup



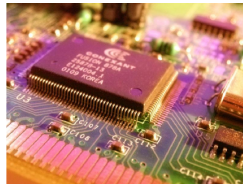
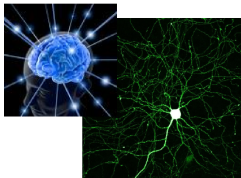
- Already successfully used for optical chaos communications
- Well-known as well in high spectral purity microwave generation
- More recently, served as the experimental basis for the two first demonstration of photonic RC
- 1<sup>st</sup> electronic demonstrator based on a similar delay dynamics
- Latest high speed photonic RC also involve delay dynamics

Brunner *et al.*, *Nature Comm.* 4:1364. Jacquot *et al.*, *CLEO Europe.* (2013)



# Time Multiplexing to address the virtual nodes

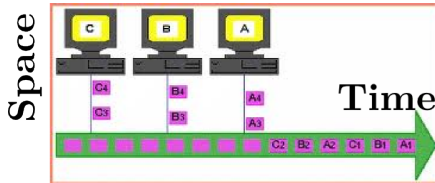
## A convenient hardware solution for RC



- Designing a complex and controlled 3D network of nodes as a brain: a very difficult technological challenge

# Time Multiplexing to address the virtual nodes

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# Time Multiplexing to address the virtual nodes

## A convenient hardware solution for RC

VOLUME 73, NUMBER 8

PHYSICAL REVIEW LETTERS

22 AUGUST 1994

### Defects and Spacelike Properties of Delayed Dynamical Systems

G. Giacomelli,<sup>1,2</sup> R. Meucci,<sup>1</sup> A. Politi,<sup>1,3</sup> and F. T. Arecchi<sup>1,4</sup>

<sup>1</sup>*Istituto Nazionale di Ottica, 50125 Firenze, Italy*

<sup>2</sup>*ITIS "Tullio Buzzi," Prato, Italy*

<sup>3</sup>*INFN, Sezione di Firenze, Firenze, Italy*

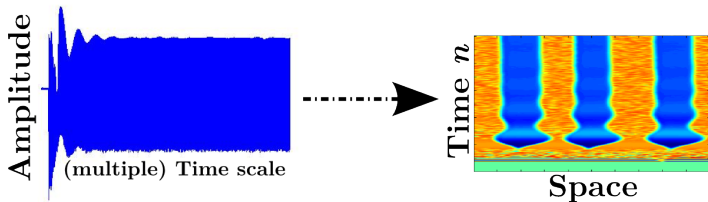
<sup>4</sup>*Dipartimento di Fisica, Università di Firenze, Firenze, Italy*  
(Received 11 January 1994)

In a laser with delayed feedback operating in an oscillatory regime, phase defects appear for delays longer than the oscillation period. These defects are visualized by rearranging the data in a two-dimensional representation. Two distinct disordered phases are observed, one of weak turbulence characterized by phase fluctuations, and one of highly developed turbulence characterized by a constant density of defects. The transition between the two regimes is analyzed by studying the dependence of the defect lifetime on the delay. The experimental findings are modeled via a generalized Landau equation which includes a delayed coupling.

- Designing a complex and controlled 3D network of nodes as a brain: a very difficult technological challenge
- Serial processing: common in many communication systems
- Delay dynamics known as virtual Space-Time dynamics

# Time Multiplexing to address the virtual nodes

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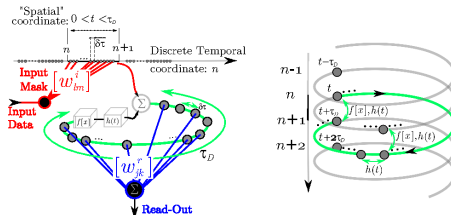


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- Feature confirmed, e.g. recently, Chimera states in delay systems

Larger, Penkovskiy, Maistrenko, "Virtual Chimera States for Delayed-Feedback Systems", *Phys.Rev.Lett.* **111** (2013)

# Time Multiplexing to address the virtual nodes

## A convenient hardware solution for RC



- Designing a complex and controlled 3D network of nodes as a brain: a very difficult technological challenge
- Serial processing: common in many communication systems
- Delay dynamics known as virtual Space-Time dynamics
- Feature confirmed, e.g. recently, Chimera states in delay systems
- Schematic of RC architecture with delay dynamics

# Outline

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Delay dynamics: a bit of theory

Basics in NL delay dynamics

Space-Time analogy, ex. of Chimera states

# Re-scaling and spatio-temporal viewpoint

**Normalization wrt Delay  $\tau_D$ :**  $s = t/\tau_D$ , and  $\varepsilon = \tau/\tau_D$

$$\varepsilon \dot{x}(s) = -x(s) + f_{\text{NL}}[x(s-1)], \quad \text{where} \quad \dot{x} = \frac{dx}{ds}.$$

Large delay case:  $\varepsilon \ll 1$ , potentially high dimensional attractor  
 $\infty$ -dimensional phase space, initial condition:  $x(s), s \in [-1, 0]$

# Re-scaling and spatio-temporal viewpoint

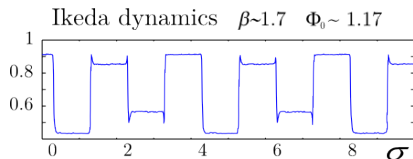
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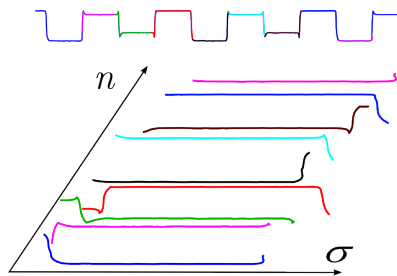
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- Discrete time  $n$

$$n \rightarrow (n+1)$$

$$s = n(1 + \gamma) + \sigma \rightarrow s = (n+1)(1 + \gamma) + \sigma$$



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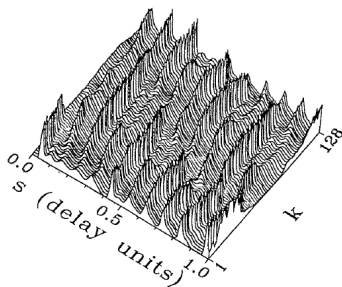
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F.T. Arecchi, *et al.* Phys. Rev. A, 1992

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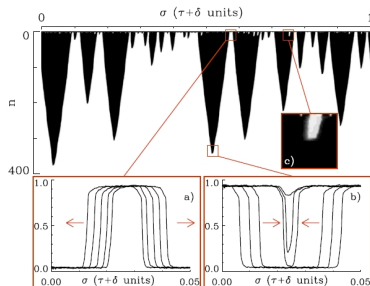
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G. Giacomelli, *et al.* EPL, 2012



# Space-Time analogy with DDE

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Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al.* *Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* **8**, 658 & 662 (2012)

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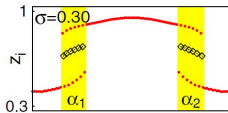


## What is a Chimera in Nonlinear Dynamics?

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- Predicted numerically in 2002, derived for a particular case in 2004, and 1<sup>st</sup> observed experimentally in 2012
- Does not exist with local coupling, neither for global one

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## Features allowing for Chimera states?

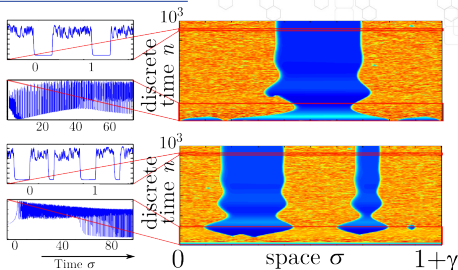
- Network of oscillators, thus spatio-temporal dynamics
- Requires non-local nonlinear coupling between oscillator nodes
- Important parameters: coupling strength, and coupling distance

Y. Kuramoto and D. Battogtokh, *Nonlinear Phenom. Complex Syst.* **5**, 380 (2002); D. M. Abrams and S. H. Strogatz, *Phys. Rev. Lett.* **93**, 174102 (2004); I. Omelchenko *et al.* *Phys. Rev. Lett.* **106** 234102 (2011); A. M. Hagerstrom *et al.* & M. Tinsley *et al.*, *Nat. Phys.* **8**, 658 & 662 (2012)

# Virtual Chimera in $(\sigma, n)$ –space

## Numerics:

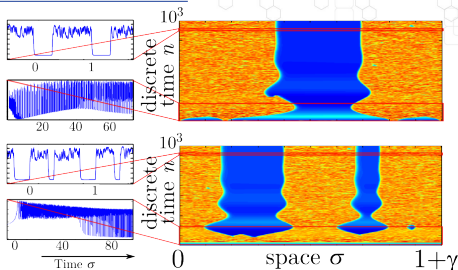
- $\beta = 0.6$ ,  $\nu_0 = 1$ ,  $\varepsilon = 5 \cdot 10^{-3}$ ,  
 $\delta = 1.6 \times 10^{-2}$  ( $m = 56$ )
- Initial conditions: small amplitude white noise (repeated several times with different noise realizations)
- Calculated durations: Thousands of  $n$



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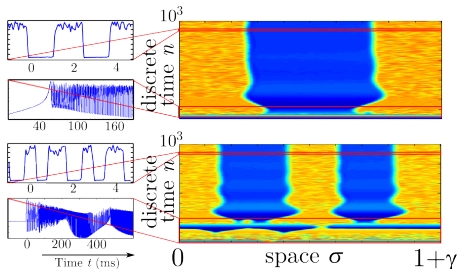
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## Experiment

- Very close amplitude and time parameters,  
 $\tau_D = 2.54\text{ms}$ ,  $\theta = 0.16\text{s}$ ,  $\tau = 12.7\mu\text{s}$
- Initial condition forced by background noise
- Recording of up to  $16 \times 10^6$  points, allowing for a few thousands





# Space-Time analogy: analytical support

---

Convolution product involving the linear impulse response,

$$h(t) = \mathbf{FT}^{-1}[H(\omega)]$$

$$x(s) = \int_{-\infty}^s h(s - \xi) \cdot f_{\text{NL}}[x(\xi - 1)] \, \mathrm{d}\xi \quad \text{with} \quad s = n(1 + \gamma) + \sigma$$

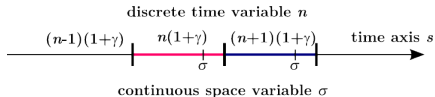
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$$]-\infty; s] = ]-\infty; n(1 + \gamma) + \sigma] \quad \cup \quad ]n(1 + \gamma) + \sigma; (n + 1)(1 + \gamma) + \sigma]$$

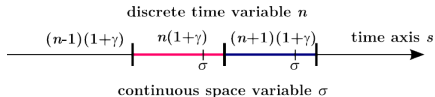
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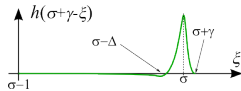
and make a change of integration variable  $\xi \leftrightarrow \xi - (n + 1)(1 + \gamma) + \gamma$

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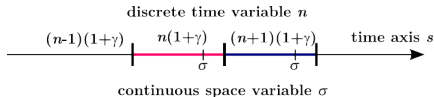
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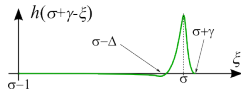
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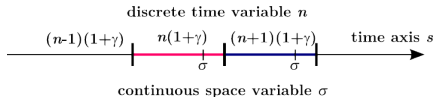
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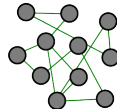


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$$\left\{ \frac{\partial \phi}{\partial t} = \omega - \int_{-\pi}^{\pi} G(x - x') \cdot \sin[\phi(x, t) - \phi(x', t) + \alpha] dx \right\}$$

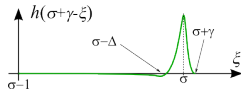


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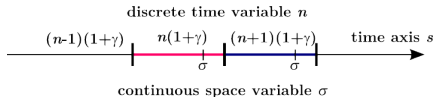
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Remark: the NL dynamics and coupling features of each virtual oscillator are by construction identical at any position  $\sigma$ !!!

# Outline

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Introduction, background, motivations & basics

RC: Where does it come from?

Important concepts in RC

Delay dynamics: a bit of theory

Photonic implementations of RC

Photonic delay-based RC for spoken digit recognition

Conclusions



Photonic implementations of RC

Photonic delay-based RC for spoken digit recognition



# Different approaches for photonic RC

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**“Actual” Spatio-temporal dynamics**

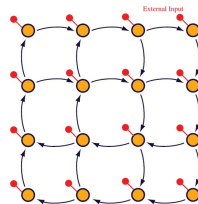


# Different approaches for photonic RC

## “Actual” Spatio-temporal dynamics

- Network of coupled SOAs (active)

Vandoorne *et al.*, *Opt.Expr.* 2008 & *IEEE Trans. Neural Network* 2011



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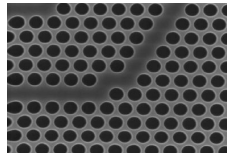
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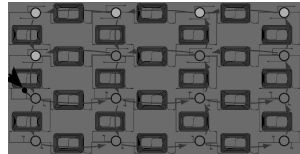
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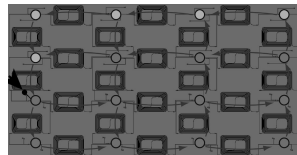
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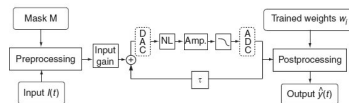
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## Emulated “virtual” through delay dynamics

- Mackey-Glass delay electronic circuit

Appeltant *et al.*, *Nature Comm.* 2011. Keuninckx *et al.* 2013



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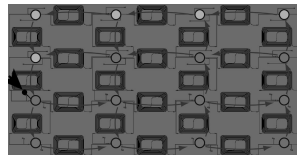
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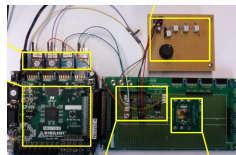
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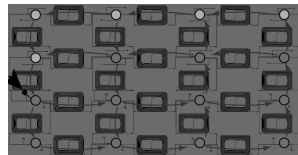
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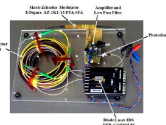
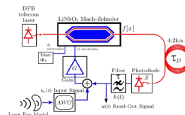
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Appeltant *et al.*, *Nature Comm.* 2011. Keuninckx *et al.* 2013

- Ikeda-like (electro-)optic delay dynamic

Larger *et al.*, *Opt.Expr.* 2012. Paquot *et al.*, *Sci.Reports* 2012

Duport *et al.*, *Opt.Expr.* 2012. . .



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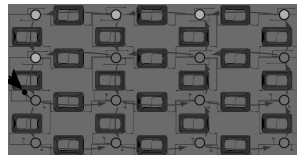
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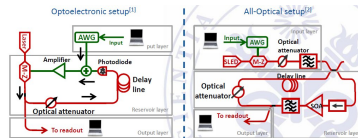
- Mackey-Glass delay electronic circuit

Appeltant *et al.*, *Nature Comm.* 2011. Keuninckx *et al.* 2013

- Ikeda-like (electro-)optic delay dynamic

Larger *et al.*, *Opt.Expr.* 2012. Paquot *et al.*, *Sci.Reports* 2012

Duport *et al.*, *Opt.Expr.* 2012. . .





# Different approaches for photonic RC

## “Actual” Spatio-temporal dynamics

- Network of coupled SOAs (active)

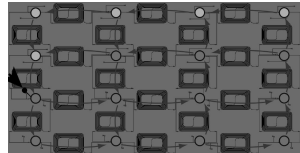
Vandoorne *et al.*, *Opt.Expr.* 2008 & *IEEE Trans. Neural Network* 2011

- Network with Photonic Crystal Structures

Fiers *et al.*, *IEEE Trans. Neur. Netw. & Learn. Syst.* 2014

- On-chip Network of coupled delay lines (passive)

Vandoorne *et al.*, *Nature Comm.* 2014



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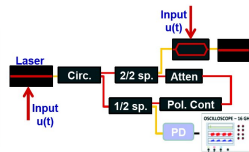
- Ikeda-like (electro-)optic delay dynamic

Larger *et al.*, *Opt.Expr.* 2012. Paquot *et al.*, *Sci.Reports* 2012

Duport *et al.*, *Opt.Expr.* 2012. . .

- External cavity Laser Diode

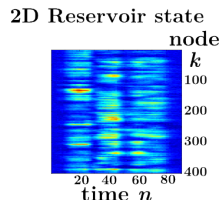
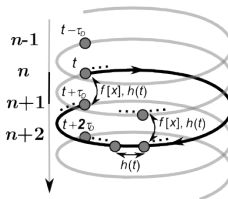
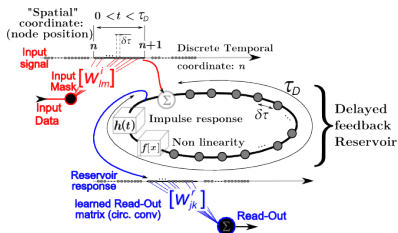
Brunner *et al.*, *Nature Comm.* 2013



# Delay Dynamics as a Reservoir



## Spatio-Temporal viewpoint of a DDE

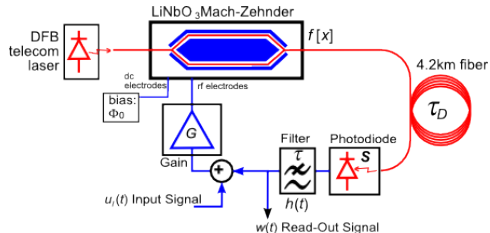
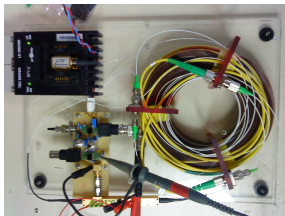


Discrete time variable: Delay time step

Virtual Spatial variable: internal delay waveform

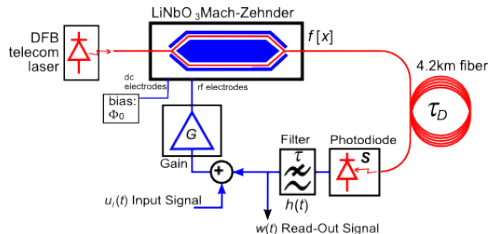
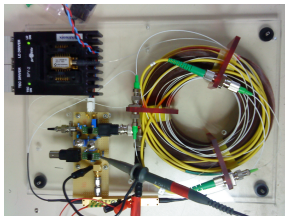
# Optoelectronic DDE as a Reservoir

## Experimental setup



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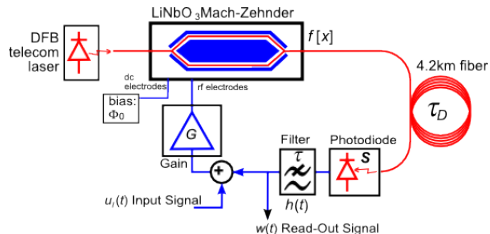
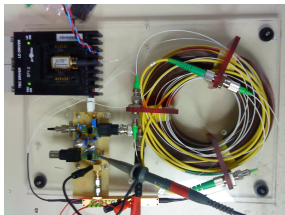


## Accurate & simple modeling

$$\varepsilon \dot{x}(s) + x(s) = \beta \sin^2[x(s-1) + \rho u_I(s-1) + \Phi_0].$$

# Optoelectronic DDE as a Reservoir

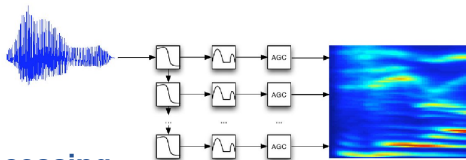
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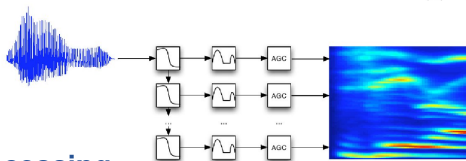
# Dynamical Processing of Spoken Digits



## Input pre-processing

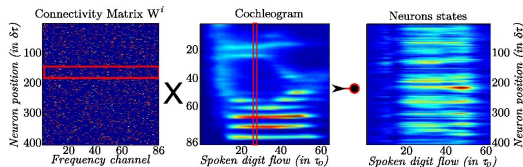
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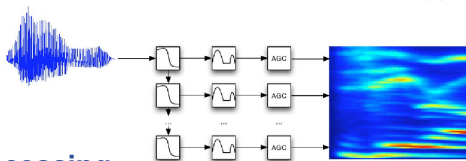


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- Sparse “connection” of the 86 Freq. channel to the 400 neurons: random connection matrix

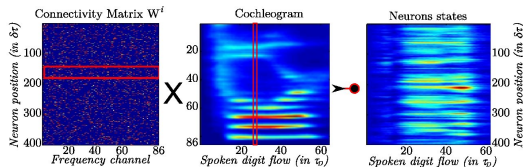


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## Reservoir transient response:

- Time series record for Read-Out post-processing



# Read-Out, Training, and Testing

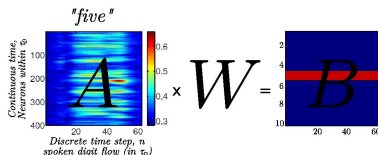


## Training of the Read-Out with target output function

Learning: optimization of the  $W$  matrix,  
for each different digit

→ Regression problem for  $A \times W \simeq B$ :

$$W_{\text{opt}} = (A^T A - \lambda I)^{-1} A^T B$$



# Read-Out, Training, and Testing

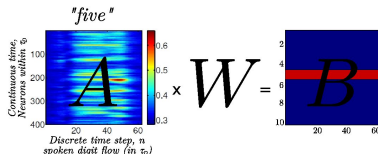


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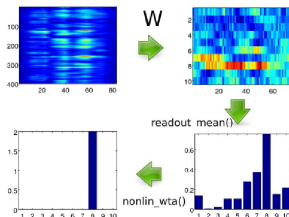
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## Testing with training-defined Read-Out

Test result: State of the art  
(close to 0% Word Error Rate)



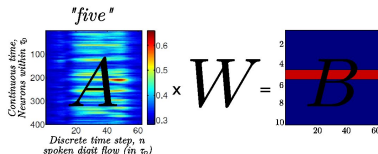
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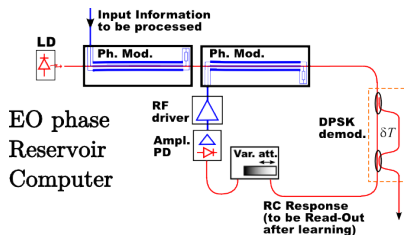
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With Telecom Bandwidth setup:  
record speed recognition, 1M word/s



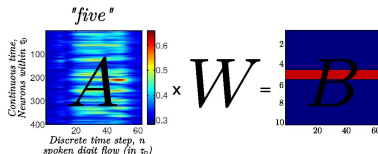
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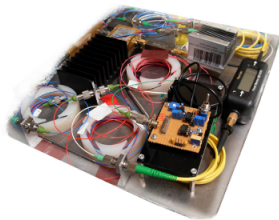
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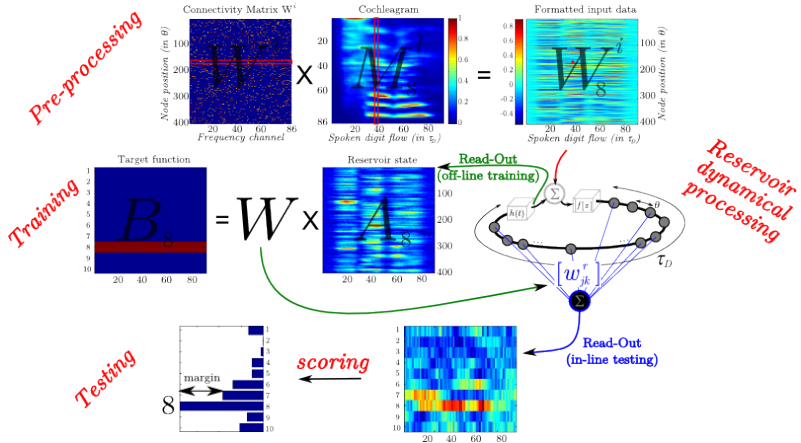
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# Example of a benchmark test with RC

## Spoken digit recognition

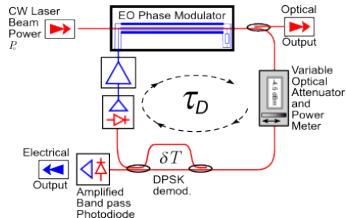
(from T146 speech corpus, 500 words, 0-9, 5x uttered, 10 speakers)



# High speed setup inspired by DPSK photonic

## Electro-optic PHASE delayed feedback Reservoir

- Dual delay setup (DPSK demodulator)

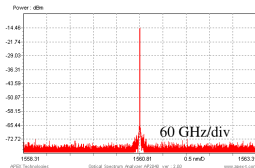
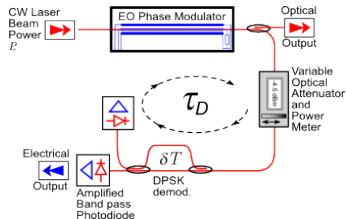


Lavrov *et al. Phys.Rev.E* **80** (2009)

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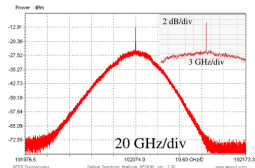
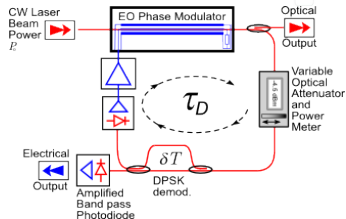


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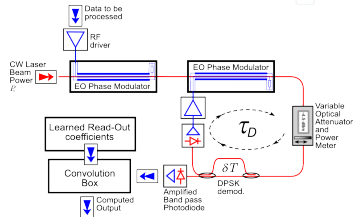
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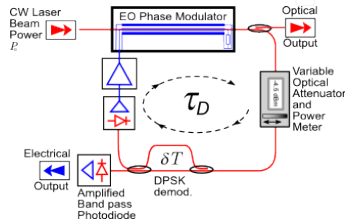


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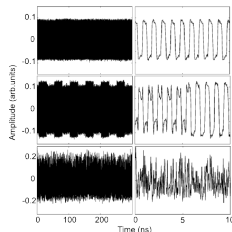
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$$\varphi(t) + \tau \frac{d\varphi}{dt}(t) = \beta \cdot \{ \cos[\varphi(t - \tau_D) - \varphi(t - \tau_D - \delta T) + \Phi_0] - \cos \Phi_0 \}$$

## Nonlinear dynamics issues

- three time scales NL dynamics ( $\tau \ll \delta T \ll \tau_D$ )
- Hopf bifurcation @  $(\beta, \Phi) = (0.5, \pi/4)$ ,  $f_H = (2\delta T)^{-2}$
- Modulation instability with period  $\tau_D$
- Period doubling-like route to chaos

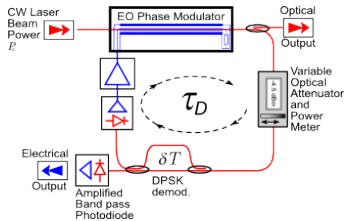


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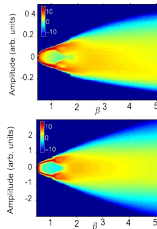
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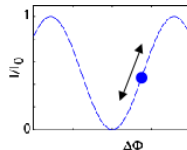


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# Million words/s speech recognition

## Operating conditions

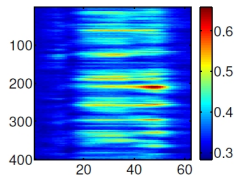
- Rest point along the nonlinear function,  $\Phi_0 \simeq 2\pi/5$
- Feedback gain  $\beta$  (edge of instability): 0.7
- Information weight (nonlinear strength):  $1.2\pi$
- Input mask sampling: 17.6 GHz (56.8 ps)
- Number of virtual nodes (neurons): 371
- unmasked input sample / delay: 3
- Average processing time per digit:  $60 \times 371 \times 56.8 \text{ ps} = 1.26 \mu\text{s}$



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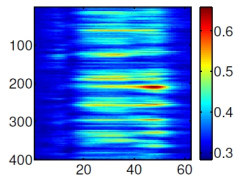
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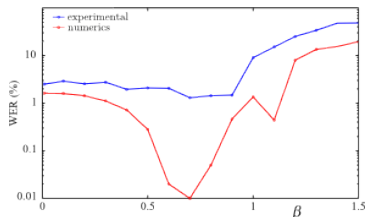
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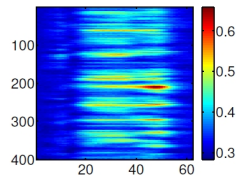
## WER performance



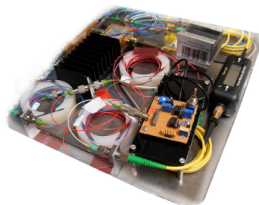
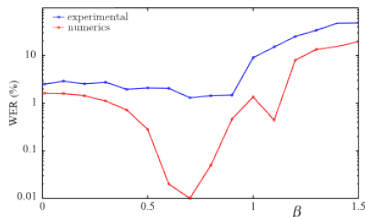
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## WER performance



# Outline

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Introduction, background, motivations & basics

RC: Where does it come from?

Important concepts in RC

Delay dynamics: a bit of theory

Photonic implementations of RC

Conclusions



# Conclusion, and perspectives

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## RC achievements



# Conclusion, and perspectives

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## RC achievements

- A novel, and contest winner, computational paradigm



# Conclusion, and perspectives

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## RC (near) future issues

- Address real-world problems (not academic benchmark)



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- ... and many other steps towards...  
the future **PRC**





# Thank you for your attention

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**INTERNATIONAL  
YEAR OF LIGHT  
2015**

# The simplest beginning in Dynamical Systems

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## Linear first order scalar dynamics

$$\tau \frac{dx}{dt}(t) + x(t) = 0, \quad \tau: \text{response time}$$
$$\dot{x} = -\gamma \cdot x, \quad \gamma = 1/\tau: \text{rate of change}$$

Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

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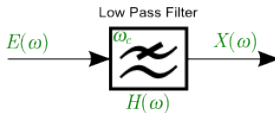
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## Time and Fourier domains (FT $\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with  $X(\omega) = \text{FT}[x(t)]$ , and  $E(\omega) = \text{FT}[e(t)]$ , &  $\omega_c = 1/\tau$



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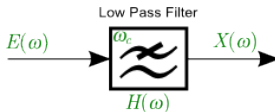
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$$(1 + i\omega\tau) \cdot X(\omega) = H_0 \cdot E(\omega) \quad \xrightarrow{\text{FT}^{-1}}$$

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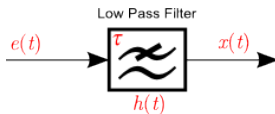
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$$(1 + i\omega\tau) \cdot X(\omega) = H_0 \cdot E(\omega) \quad \xrightarrow{\text{FT}^{-1}} \quad x(t) + \tau \frac{dx}{dt}(t) = H_0 \cdot e(t)$$

(remember  $\text{FT}^{-1}[i\omega \times (\cdot)] = \frac{d}{dt}\text{FT}^{-1}[(\cdot)]$ )

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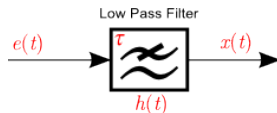
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Simplest modeling of the un-avoidable continuous time (finite speed, or rate) physical transients

## Time and Fourier domains (FT $\equiv$ Fourier Transform)

$$H(\omega) = \frac{H_0}{1+i\omega\tau} = \frac{X(\omega)}{E(\omega)}$$

with  $X(\omega) = \text{FT}[x(t)]$ , and  $E(\omega) = \text{FT}[e(t)]$ , &  $\omega_c = 1/\tau$



$$(1 + i\omega\tau) \cdot X(\omega) = H_0 \cdot E(\omega) \quad \xrightarrow{\text{FT}^{-1}} \quad x(t) + \tau \frac{dx}{dt}(t) = H_0 \cdot e(t)$$

(remember  $\text{FT}^{-1}[i\omega \times (\cdot)] = \frac{d}{dt}\text{FT}^{-1}[(\cdot)]$ )

$$h(t) = \text{FT}^{-1}[H(\omega)] \quad [(\text{causal}) \text{ impulse response}], \quad \rightarrow \quad x(t) = \int_{-\infty}^t h(t-\xi) \cdot e(\xi) d\xi$$

# Solutions, initial conditions, phase space

**Autonomous case** ( $e(t) = e_0, \Leftrightarrow e \equiv 0$  with  $z = x - e_0$ )

$$\begin{aligned} \tau \dot{x} + x &= 0, & 0: \text{(dead) fixed point } (\dot{x} = 0) \\ \Rightarrow x(t) &= x_0 e^{-t/\tau} = x_0 e^{-\gamma t}, & \gamma: \text{convergence rate} \rightarrow 0, \forall x_0 \end{aligned}$$

$-\gamma : < 0$  eigenvalue (stable);      Size of the init. cond.,  $\dim x_0 = 1 \Rightarrow 1\text{D dynamics (or phase space)}$

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**Feedback ( $e(t) = f[x(t)]$ ): stability, multi-stability**

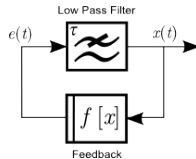
Fixed point(s):  $\{x_F \mid x = f[x]\}$

(Graphics: intersect(s) between  $y = f[x]$  and  $y = x$ )

Stability @  $x_F$ : linearization for  $x(t) - x_F = \delta x(t) \ll 1$ ,

$$f[x] = x_F + \delta x \cdot f'[x_F] \Rightarrow \dot{\delta x} = -\gamma(1 - f'_{x_F}) \cdot \delta x = \gamma_{fb} \cdot \delta x$$

$f'_{x_F} < 0 \equiv \text{P-negative feedback, speed up the rate; } f'_{x_F} > 0, \text{ slow down the rate, possibly unstable if } > 1$





# Solutions, initial conditions, phase space

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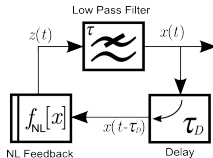
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$f'_{x_F} < 0 \equiv \text{P-negative feedback, speed up the rate; } f'_{x_F} > 0$ , slow down the rate, possibly unstable if  $> 1$



## Delayed feedback ( $e(t) = f[x(t - \tau_D)]$ ): $\infty$ -dimensional

Fixed point(s):  $\{x_F \mid x = f[x]\}$

Stability:  $\delta x(t) = a \cdot e^{\sigma t}$ , eigenvalues:  $\{\sigma \in \mathbb{C} \mid 1 + \sigma\tau = e^{-\sigma\tau_D} \cdot f'_{x_F}\}$ ,

Size of initial conditions:  $\{x(t), t \in [-\tau_D; 0]\} \Rightarrow \infty\text{D phase space}$