Indexing, Storing and Retrieving Data in Neural Networks

Vincent Gripon

July 1st, 2017

Outline

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1. Computer Vision and Neural Networks
2. Hopfield Neural Networks
3. Willshaw Neural Networks
4. Conclusion

Computer vision problems

Vector space ($\mathbb{R}^d$)

1. Supervised learning,
2. Unsupervised learning,
3. Indexing,
4. Search,
5. ...

Computer vision problems

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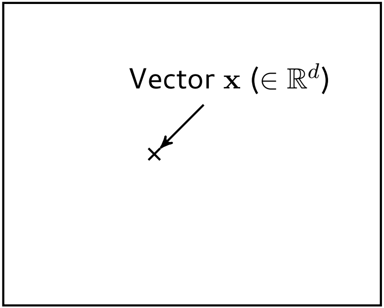

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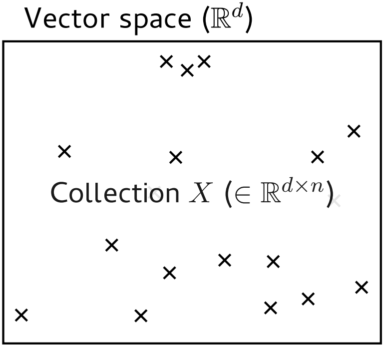

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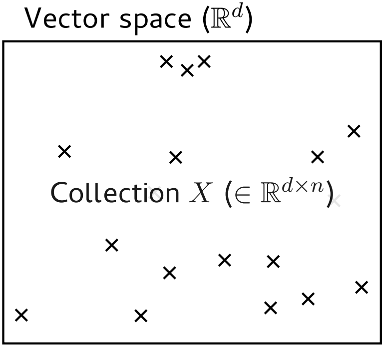

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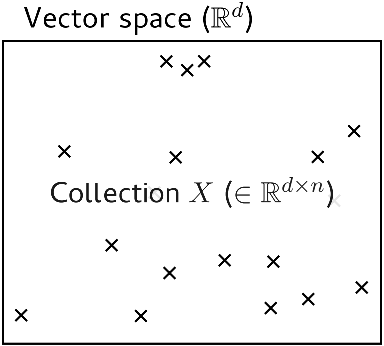

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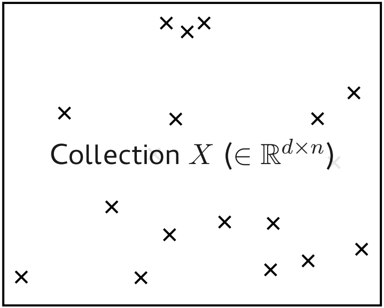

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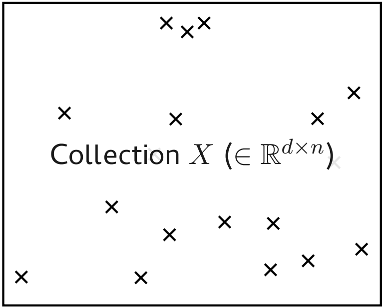

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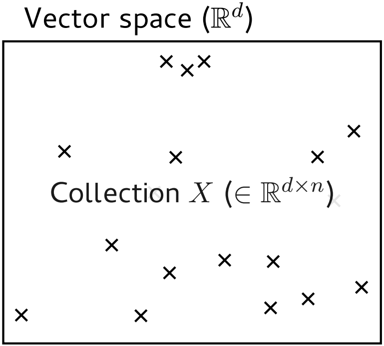

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Relation with images

Relation with images

0.88461	0.52899	0.39796	0.156	0.22615	0.16447	0.84366	0.7841	0.05846	0.3335	0.04146	0.79913
0.33711	0.22133	0.46954	0.8197	0.97514	0.79205	0.19736	0.33366	0.05208	0.04472	0.23042	0.93124
0.8572	0.45831	0.87231	0.71693	0.63678	0.54683	0.24892	0.32603	0.82655	0.08347	0.76076	0.59149
0.33481	0.62067	0.78087	0.55115	0.82832	0.46957	0.5429	0.7357	0.49622	0.09038	0.59702	0.38432
0.7534	0.19463	0.41368	0.23335	0.01205	0.18668	0.9122	0.00722	0.64043	0.78145	0.94182	0.77094
0.45204	0.64851	0.0368	0.38763	0.99484	0.14494	0.76273	0.27692	0.33253	0.70724	0.7361	0.36882
0.35962	0.0953	0.47678	0.92337	0.72545	0.3611	0.05582	0.48013	0.5318	0.27792	0.90964	0.15971
0.528	0.4521	0.6933	0.3117	0.57884	0.00188	0.06187	0.60576	0.94542	0.62769	0.82405	0.40215
0.4817	0.3089	0.50847	0.56479	0.91013	0.38911	0.1955	0.19717	0.80548	0.0926	0.54935	0.2212
0.2007	0.39793	0.76196	0.40977	0.5557	0.13638	0.11624	0.72516	0.711	0.37856	0.34254	0.67796
0.18808	0.495	0.61931	0.85258	0.15338	0.95236	0.7579	0.83098	0.89072	0.30334	0.79318	0.93652
0.73792	0.10391	0.66104	0.11888	0.31796	0.11823	0.3503	0.21704	0.67531	0.10696	0.15614	0.88287
0.87881	0.5232	0.7498	0.49826	0.56987	0.82922	0.50221	0.17014	0.14153	0.50203	0.71329	0.5883
0.82059	0.15565	0.77045	0.65742	0.69325	0.81161	0.6689	0.2689	0.3157	0.30891	0.10176	0.50745
0.35197	0.55392	0.10223	0.35126	0.41571	0.52171	0.06691	0.90193	0.5367	0.32317	0.99646	0.32294
0.22276	0.84074	0.81502	0.19206	0.1831	0.95694	0.19064	0.521	0.00731	0.2918	0.75076	0.04846
0.83913	0.02509	0.23013	0.56805	0.48898	0.59413	0.00085	0.5568	0.94305	0.58304	0.97273	0.23122
0.8672	0.99681	0.55971	0.3113	0.93437	0.85008	0.31642	0.69461	0.92604	0.10722	0.9894	0.79062
0.73033	0.55927	0.65729	0.24986	0.83891	0.9922	0.34003	0.88657	0.22913	0.68306	0.26389	0.36389
0.7484	0.26044	0.84335	0.92747	0.1045	0.4189	0.38507	0.61227	0.37956	0.96606	0.76146	0.66606
0.80537	0.69485	0.216	0.07784	0.07887	0.0033	0.78531	0.68555	0.85399	0.55963	0.3055	0.38026
0.58958	0.92455	0.1638	0.98196	0.19221	0.06602	0.34438	0.62881	0.6117	0.7972	0.42224	0.32251
0.91573	0.59866	0.83585	0.58174	0.16328	0.43048	0.93353	0.46086	0.84715	0.23059	0.77397	0.77893
0.9289	0.79767	0.2249	0.59573	0.66231	0.51534	0.836	0.75488	0.11812	0.52908	0.59819	0.46626
0.62892	0.8288	0.5407	0.22375	0.89394	0.67827	0.91756	0.87721	0.45978	0.0395	0.76498	0.70155

Relation with images

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1. Features extraction
2. Features comparison

25% error rate in 2011

Supervised learning

Learning

To learn is to **generalize** ($\neq$ memorize),

Supervised learning

- Regression,
- Requires an expert,
- Has a lot of applications:
  - Playing games,
  - Pattern recognition,
  - Attending to a lesson...

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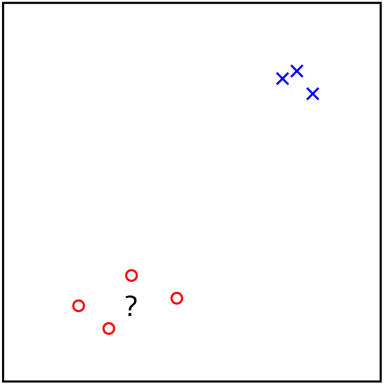

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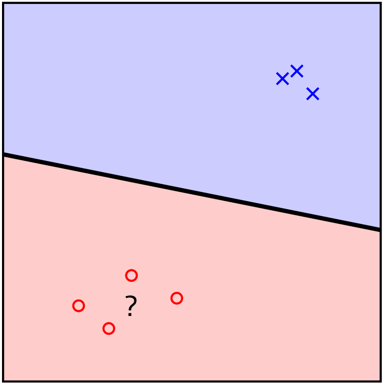

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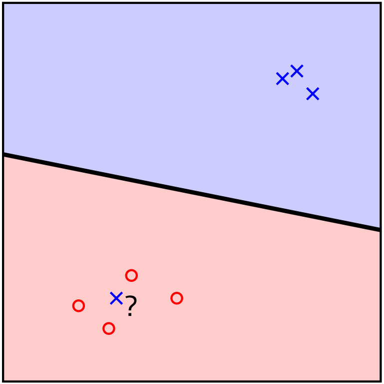

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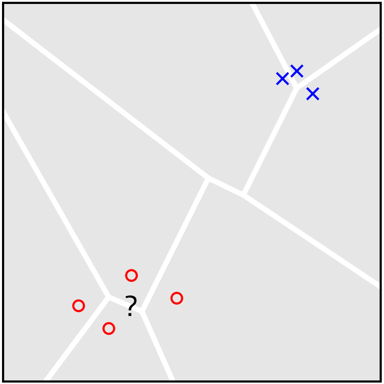

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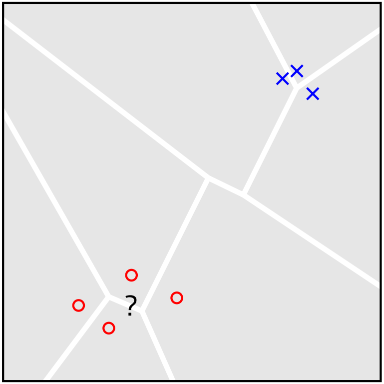
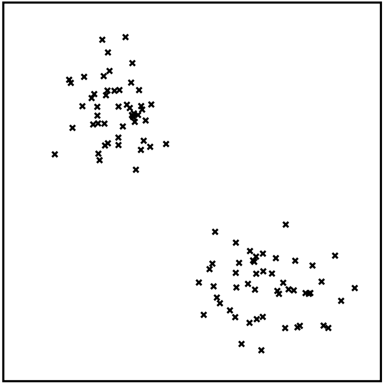


Classical methods: SVM, k-NN, Random Forests, LR, MLP, CNN...

Unsupervised learning



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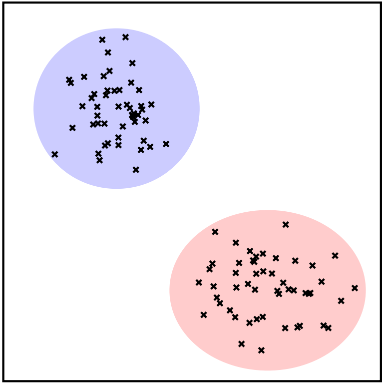

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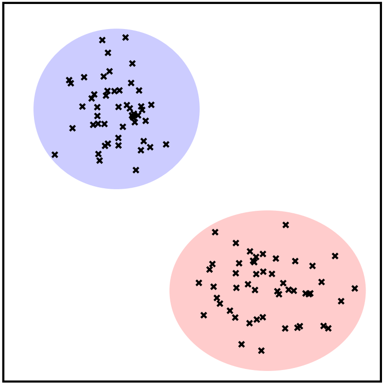

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Classical methods: k-means, db-scan, Kohonen maps, autoencoders, EM...

Indexing

Definition

Given a collection $X \in \mathbb{R}^{d \times n}$ and a query vector $\mathbf{x}$:

1. Is $\mathbf{x} \in X$?
2. Do we have $\mathbf{x}' \in X$ s.t. $\mathbf{x}' \approx \mathbf{x}$?

Exhaustive search

- Pros: no error, simple, concurrent,
- Cons: linear with both d and n.

Example of database: SIFT1B:

- $n = 1,000,000,000$,
- $d = 128$,
- 10,000 tests,
- On my laptop, takes approximatively 4 years.

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Methods

Hash tables (exact)

- Store items in an array of lists,
- Access array addresses using hash functions.

Bloom filters (approximate)

- Retain already-seen hashes,
- Compare with probed ones.

Locality Sensitive Hashing (LSH)

- Use smooth hashes,
- Convert Euclidean to Hamming.

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Given a collection $X \in \mathbb{R}^{d\times n}$ and a query vector $\mathbf{x}$, find:

$$\mathbf{x}' = \arg\min_{\mathbf{x}'\in X} \|\mathbf{x} - \mathbf{x}'\|,$$

given some metric.

Methods

- Exhaustive search again,
- Act on n and/or d:
  - On n, partition the search space (problems with high dimensions),
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Curse of dimensionality

"Intuition is wrong in high dimension."

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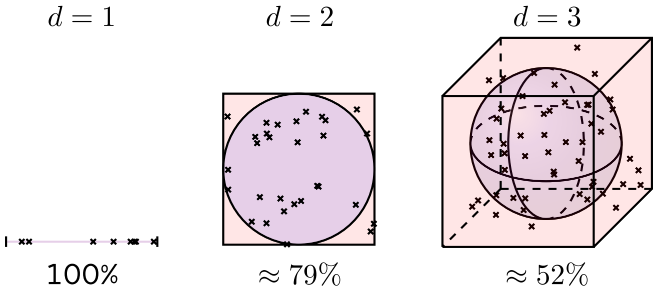

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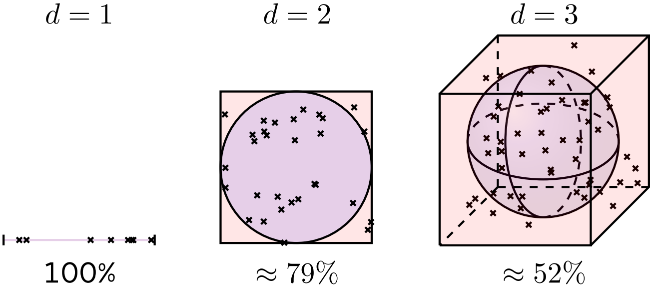

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A neural network performs iteratively the concatenation of a linear and a nonlinear function.

$$\mathbf{y} = h_1(W_1(h_2(W_2 \dots h_i(W_i\mathbf{x})))).$$

Nonlinear functions

- Sigmoids (e.g. $x \mapsto 1/\left(1+\exp(-x)\right)$),
- Relus (e.g. $x \mapsto \max(0, x)$),
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A note on Neural Networks methods for vision

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- $\mathbf{x} \in \{-1,1\}^d$, $X \subset \{-1,1\}^{d \times n}$,

- Example:
  - Storing binary message -11-111-1-11
  - Retrieve it from -11-111-1?1
- $W = \sum_{\mathbf{x} \in X} \mathbf{x}\mathbf{x}^\top - diag(\mathbf{x}\mathbf{x}^\top) = XX^\top - diag(XX^\top)$,
- $\mathbf{y} = sgn(W\mathbf{x}) = u(\mathbf{x})$,
- The update can be sequential (one coordinate at a time),
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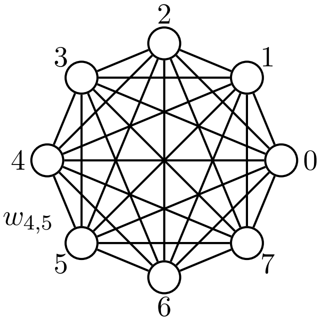

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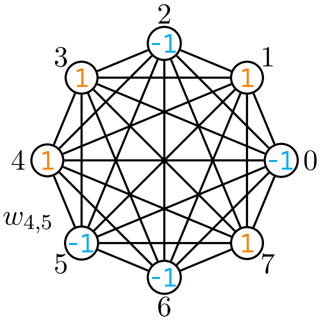

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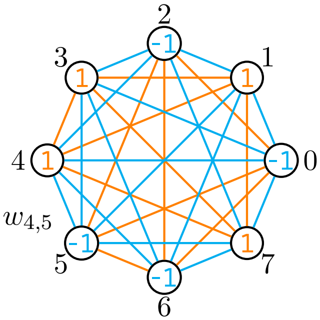

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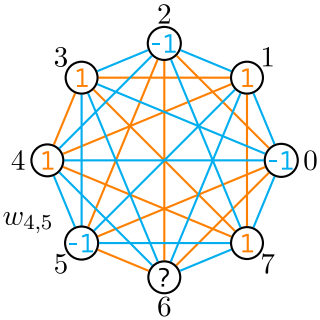

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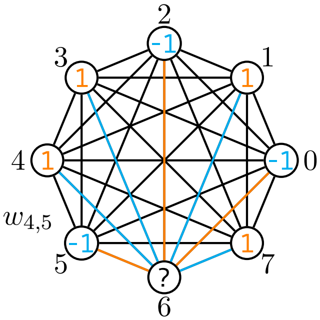

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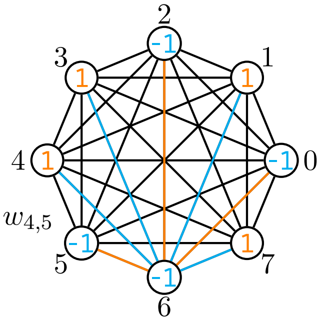

Hopfield Neural Networks

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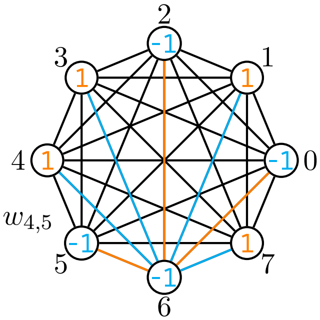

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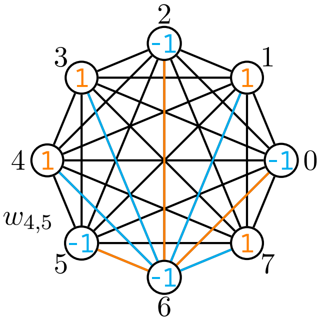

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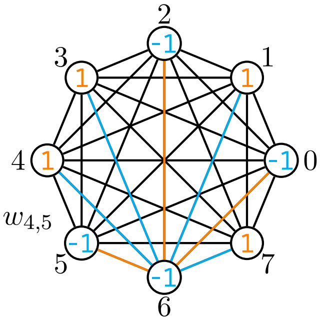

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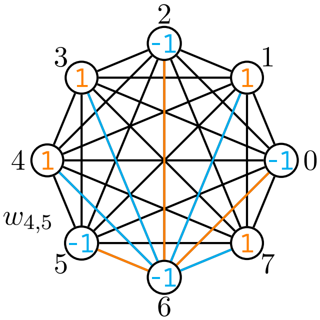

Stability of stored vectors

Theorem [1]

Consider $n = \frac{d}{\gamma \log(d)}$:

- If $\gamma > 6$, then for $d \to \infty$, $\mathbb{P}[\liminf_d \{\cap_{\mathbf{x} \in X} \{U(\mathbf{x}) = \mathbf{x}\}] = 1$,
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Memory efficiency

- $\binom{d}{2}$ connections with $n+1$ possible values each $\Rightarrow$ takes $\binom{d}{2} \log_2(n+1)$ bits without compression,
- To be compared to the entropy of $X \approx nd$ (Why $\approx$?).
- When patterns are stable, we obtain $\eta \leq \frac{1}{2\log(d)\log_2(n+1)}$.

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Consider $\rho \in [0, 1/2[$ and $n = (1-2\rho)^2 \frac{d}{\gamma \log(d)}$. $\tilde{\mathbf{x}}$ is such that it contains at most ρd symbols different from $\mathbf{x}$, then:

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The quantity $H(\mathbf{x}) = -\frac{1}{d}\sum_{i,j=1}^{d} W_{ij}\mathbf{x}_i\mathbf{x}_j$ is nonincreasing with u. More generally, $U(\mathbf{x})$ is a local minimum for H.

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Demonstration

1. Create network,
2. Test stability,
3. Test retrievability.

Outline

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- $\mathbf{x} \in \{0,1\}^d$, $\|\mathbf{x}\|_0 \ll d$, $X \subset \{0,1\}^{d \times n}$.

- Example:
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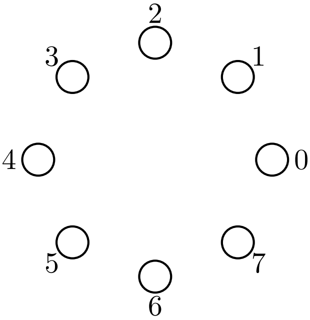

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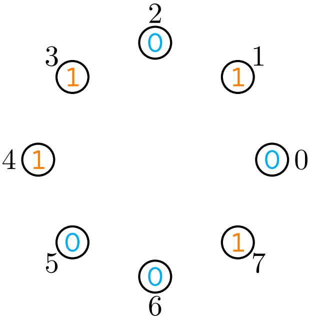

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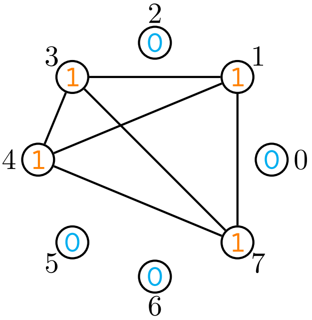

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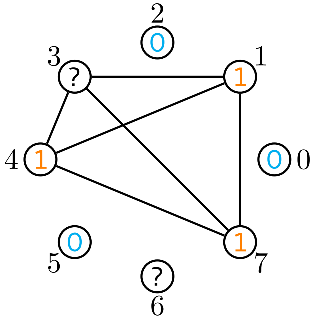

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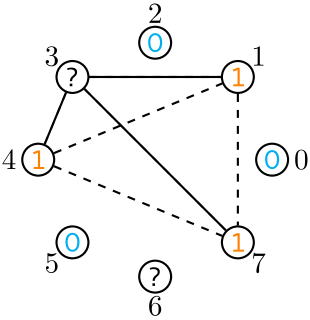

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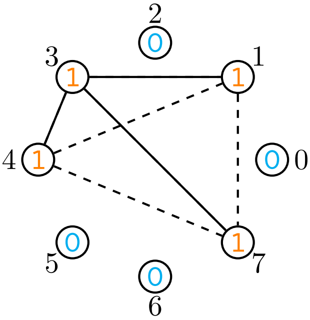

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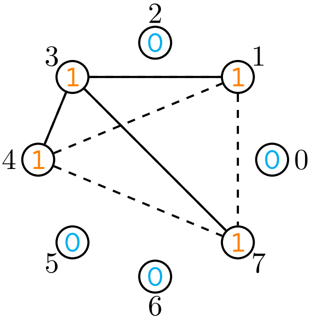

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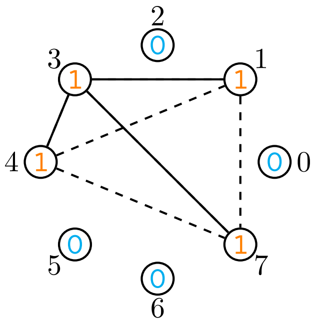

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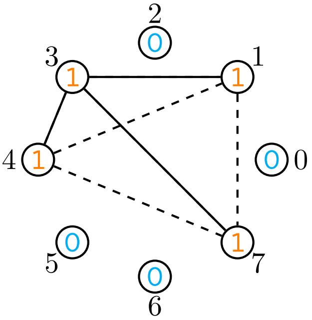

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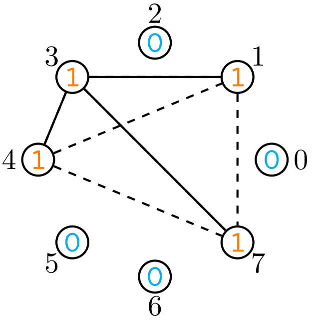

Stability of stored vectors

Theorem [2]

Consider X generated with $\|\mathbf{x}\|_0 = \lfloor \log(d)/d \rfloor$ and $\mathbf{x}$ chosen at random such that $\|\mathbf{x}\|_0 = \lfloor \log(d)/d \rfloor$. With $n = \alpha d^2 \log\log(d)/\log^2(d)$:

- If $\alpha > 2$, $\mathbb{P}[u(\mathbf{x}) = \mathbf{x}] \to 1$,
- If $\alpha = 2$, $\exists \gamma > 0$, for d large enough, $\mathbb{P}[u(\mathbf{x}) = \mathbf{x}] \geq \gamma$,
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Memory efficiency

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- $\binom{d}{2}$ connections with 2 possible values each $\Rightarrow$ takes $\binom{d}{2}$ bits without compression,
- To be compared to the entropy of X:

$$\approx ndH_2(\log(d)/d).$$

- When patterns are stable, we obtain

$$\eta \geq \frac{\alpha d(\log\log(d))^2}{2\log(d)} \to +\infty$$

Why?

Retrievability of stored vectors

Theorem [2]

Consider $n = \alpha d^2 / \log^2(d)$, $\rho \in [0, 1[$ such that $\lfloor \rho \log(d) \rfloor$ of 1s in $\mathbf{x}$ are erased to obtain $\tilde{\mathbf{x}}$. Then:

- If $\alpha < -\log(1 - \exp(-1/(1-\rho)))$, then $\mathbb{P}[u(\tilde{\mathbf{x}}) = \mathbf{x}] \to 1$,
- If $\alpha > -\log(1 - \exp(-1/(1-\rho)))$, then $\mathbb{P}[u(\tilde{\mathbf{x}}) \neq \mathbf{x}] \to 1$.

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Demonstration

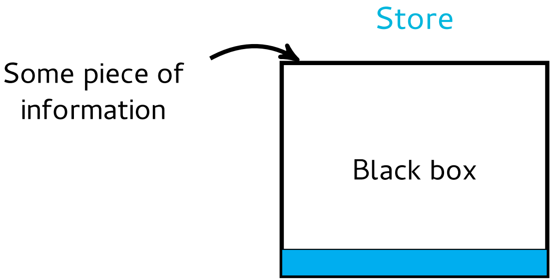
1. Create network,
2. Test stability,
3. Test retrievability.

Outline

Store

Black box

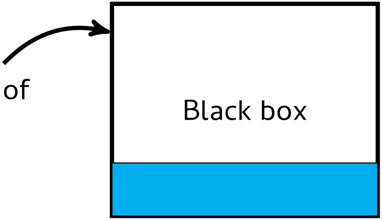
Associative memories



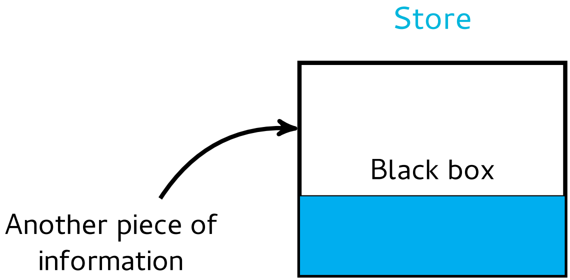
Associative memories

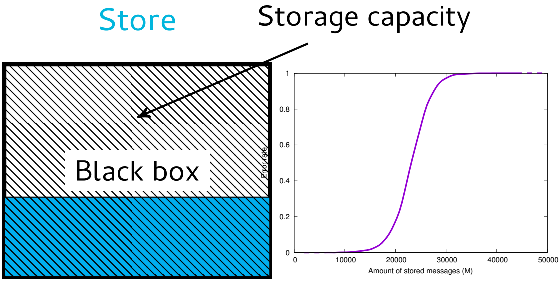
Store

Another piece of information

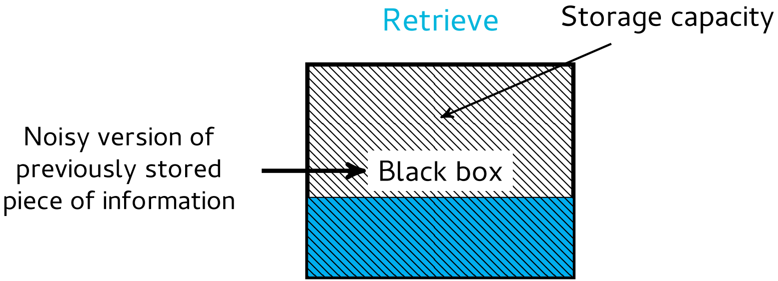


Associative memories




Store
Storage capacity
Black box
Error rate
0
0.2
0.4
0.6
0.8
1
0
10000
20000
30000
40000
50000
Amount of stored messages (M)

Associative memories



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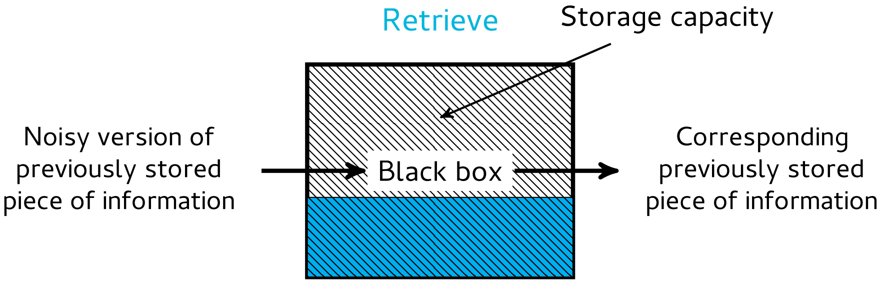

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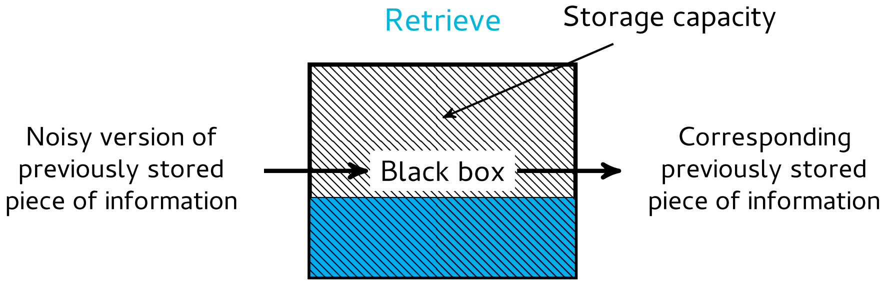

Piece of information = message, retrieve = decode
associative memory = universal decoder

Summary

	Hopfield	Willshaw
Framework	$\mathbf{x} \in \{-1,1\}^d$	$\mathbf{x} \in \{0,1\}^d$, $\|\mathbf{x}\|_0 \ll d$
Memory	$\mathbf{x}\mathbf{x}^\top - diag(\mathbf{x}\mathbf{x}^\top)$	$\mathbf{x}\mathbf{x}^\top$
Aggregation	$W = \sum_{\mathbf{x}\in X} \mathbf{x}\mathbf{x}^\top - diag(\mathbf{x}\mathbf{x}^\top)$	$W = \max_{\mathbf{x}\in X} \mathbf{x}\mathbf{x}^\top$
Search	$u(W \cdot \tilde{\mathbf{x}})$	$u(W \otimes \tilde{\mathbf{x}})$

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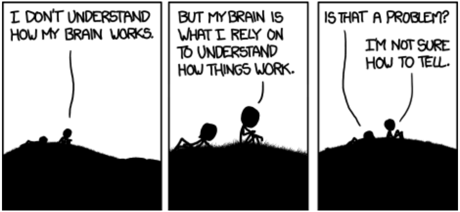
Conclusion/Take home message

- Neural networks can do much more than learning,
- Neural networks are not just big mathematical functions,
- Storing and indexing boils down to Gram matrices and strange algebras.



XKCD

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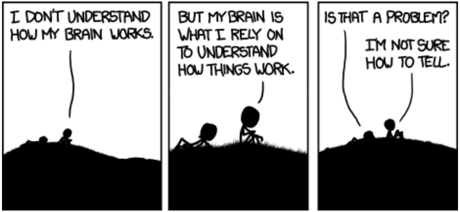

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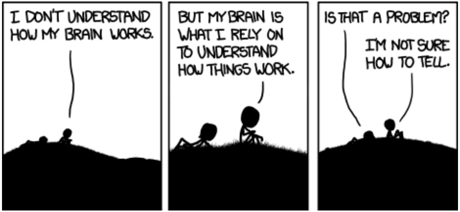

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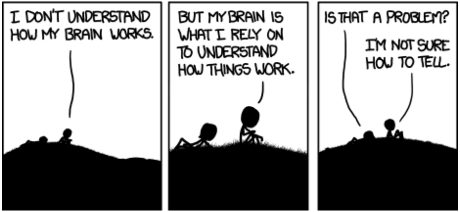

XKCD